The First Mongolia-Russia-Vietnam Workshop on Numerical Solution of Differential and Integral Equations (NSDIE 2016)

PROGRAM

&

ABSTRACTS

September 10-11, 2016

Ulaanbaatar, Mongolia

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Institute of Mathematics and Department of Applied Mathematics of National University of Mongolia

German-Mongolian Institute for Resources and Technology

Matrosov Institute for System Dynamics and Control Theory of Siberian Branch of Russian Academy of Sciences

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Conference place

German-Mongolian Institute for Resources and Technology (GMIT)
Main Building

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WORKSHOP PROGRAM

September 10, 2016

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11:20-12:00	Nguyen Khoa Son (Vietnam National University, Vietnam) *Robustness of Controllability of Linear Systems with Constrained Controls under Structured Perturbations*	
12:00-12:40	Anatoly Apartsyn (Melentiev Energy Systems Institute SB RAS, Russia) *Nonclassical Volterra equations of the first kind*	
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Session

German-Mongolian Institute for Resources and Technology Main Building

Time	Speaker and title	Chair
14:30-14:45	Vu Ngoc Phat and Nguyen Huyen Muoi. *Global finite-time stabilization of linear differential-algebraic equations with time-varying delay*	Vu Ngoc Phat
14:45-15:00	Gantulga Tsedendorj, Selenge Tsend-Ayush and Hiroshi Isshiki. *Numerical study of two-dimensional advection-diffusion problem in an infinite domain*	
15:00-15:15	Elena Chistyakova and Victor Chistyakov *The Index Preservation Property for Differentiable DAEs*	
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16:15-16:30	Mikhail Bulatov *Matrix polynomials and their application*	
16:30-16:45	Enkhbat Rentsen, Bayanjargal Darkhijav and Tungalag Nazagdorj *Application of Stochastic Differential Equations in Population Growth*	
16:45-17:00	Liubov Solovarova. *On self-regularization properties of difference scheme for linear differential-algebraic equations*	
17:00-17:15	Barsbold Bazarragchaa *Reducibility of linear homogeneous system differential equations with semiperiodic coefficients*	
17:15-17:30	Ulziibayar Vandandoo *The necessary and sufficient convergence conditions for some two and three−point Newton's type iterations*	
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ABSTRACTS

Application of Stochastic Differential Equations in Population Growth

Enkhbat Rentsen, Bayanjargal Darkhijav and Tungalag Nazagdorj
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ABSTRACT

Population as a key macroeconomic indicator plays an important role in decision making, social security and economic growth. It is important to predict population growth using various mathematical models. Population is as a function of time usually described by dynamic models based on differential equations. The most common practical methods are component and exponential. On the other hand, the population growth can be considered as stochastic variables since population depends on social and economic policies, political stability, epidemic decrease, plague, disaster and so on. In this talk, we first examine the existing population models such as component and exponential. Then we consider a stochastic model for population growth. Using the proposed model, we predicted Mongolian population growth up to 2035 year.

Keywords: Differential equation, growth theory simulation, population models.

A Model for Pension Insurance Indicators of Mongolia by Differential Equations

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National University of Mongolia (Mongolia)

ABSTRACT

The talk is devoted to a comprehensive research aimed at introducing a new national pension scheme called Basic Pension Insurance in relation with pension reform in Mongolia. Results of research consisted of basic survey and estimation justifying the Basic Pension Insurance such as determination of life-expectancy of Mongolian people at retirement, forecasting of minimum living standard, the minimum and average wages, and population of Mongolia. We constructed a mathematical model for estimating pension fund and pension expenditure based on differential equations.

Keywords: Basic pension insurance, actuarial mathematics, Life expectancy, Pension fund and Pension expenditure.

Global Finite-Time Stabilization of Linear Differential-Algebraic Equations with Time-Varying Delay

Vu Ngoc Phat and Nguyen Huyen Muoi
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ABSTRACT

Finite-time stabilization problem consists of the design of state feedback controllers such that the closed-loop system is finite-time stable [1,2]. Most of the results in the literature are focused on the Lyapunov stabilization, while few investigations have been taken on the finite-time stabilization. In this paper, we develop a general framework for global finite-time stabilization of linear differential-algebraic equations with time-varying delay. Based on Lyapunov-like function method and new bound estimation technique, we provide sufficient conditions for global finite-time stabilization of such equations.The proposed conditions given in terms of linear matrix inequalities allow us to construct the state feedback controllers, which can be easily determined by utilizing MATLABs LMI Control Toolbox [3]. A numerical example is given to illustrate the efficiency of the proposed methods.

Keywords: Finite-time stabilization, differential-algebraic equations, feedback controller, time-varying delay, linear matrix inequalities.

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On the vector characteristic exponents of the solutions of Differential-Algebraic Equations and Implicit Difference Equations

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Phan Thi Tuyet
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Mikhail Bulatov
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ABSTRACT

The concept of vector characteristic exponent was introduced by Hoang Huu Duong [1] for studying the stability properties of the ordinary differential equations. In the first part of this talk, we develop this concept for a linear differential algebraic equations with properly stated leading term. The concept of vector characteristic exponent was introduced by Doan Trinh Ninh for studying the stability properties of the ordinary difference equations. In the second part of this talk, we develop this concept for the linear implicit difference equations.

Keywords: Vector characteristic exponent, ordinary differential equations, difference equations, linear differential algebraic equations with properly stated leading term, linear implicit difference equations.

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Robustness of Controllability of Linear Systems with Constrained Controls under Structured Perturbations

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Do Duc Thuan
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ABSTRACT

In this paper, the robust controllability for linear systems with constrained controls $\dot{x} = Ax + Bu, u \in \Omega$, is studied under the assumption that Ω is an arbitrary subset satisfying the only condition $0 \in \mathrm{cl\,co}\,\Omega$ and the system matrices A, B are subjected to structured perturbations : $[A, B] \to [A, B] + D\Delta E$. The notion of the structured local and global controllability radii are defined. Based on properties of convex processes, some general formulas for the complex and the real controllability radii, for both local and global controllability concepts, are established with respect to structured affine perturbations and multi-perturbations of matrices *A*, *B*. As particular cases, the general results are applied to linear systems with bounded controls and single-input linear systems, yielding some explicit and computable formulas of controllability radii. Examples are given to illustrate the obtained results.

Keywords: Linear system, constrained control, recession cone, convex process, structured perturbation, controllability radius.

Numerical Simulations of Stochastic Neural Field Equations with Delays

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ABSTRACT

Neural Field Equations (NFE) are a powerful tool for analysing the dynamical behaviour of populations of neurons. The analysis of such dynamical mechanisms is crucially important for understanding a wide range of neurobiological phenomena. As in other sciences, in Neurobiology it is well-known that better consistency with some phenomena can be provided if the effects of random processes in the system are taken into account. The main goal of the present work is to analyze the effect of noise in certain neural fields with delay. We apply a numerical scheme which uses the Galerkin method for the space discretization and obtain a system of stochastic delay differential equations which are then discretized by the Euler-Maruyama method. We use this computational algorithm to analyze noise induced changes in the dynamical behaviour of some neural fields. Some numerical examples are presented and the results are discussed.

Keywords: Neural field equation, Euler-Maruyama method, Galerkine method.

Nonclassical Volterra Equations of the First Kind

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ABSTRACT

The report provides an overview of the results obtained by the author in the field of nonclassical Volterra equations of the first kind. The report consists of two parts. The first part is devoted to equations that arise under modeling of nonlinear input-output dynamic systems by Volterra polynomials. Special attention is given to polynomial (multilinear) Volterra equations of the first kind. In the theory of such equations the important role is played by the Lambert function which introduced into wide practice by developers of computer algebra system MAPLE. The main results of the theory and numerical methods for solving equations discussed in the first part are published in [1-9]. The second part is devoted to nonclassical linear Volterra equations of the first kind describing the processes of aging and replace elements in developing systems. The study of the mechanism of possible loss of the solutions stability of the corresponding equations leads to use of Lambert function again. Publications [10-14] devoted to this topic.

This work has been supported by the Russian Foundation for Basic Research (Project No. 15-01-01425).

Keywords: Volterra equations of the first kind, nonlinear dynamic systems, Lambert function

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On The Modeling of Long-Term Strategies for the Development of the Electric Power Systems

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ABSTRACT

In [1], [2] the optimization problem of equipment lifetime of power stations based on one-sector variant of models of developing Glushkov type systems [3] was considered. In [4-6] the new integral model of developing system, which describes, in particular, the dynamics of aging and replacement of power plants equipment, was proposed. Elements of the theory of corresponding Volterra equations of the first kind are presented in [7]. In this paper we consider the vector model of electric power system of Russia. Here generating capacities are divided into three types: thermal power plants, nuclear power plants and hydro power plants respectively. The plants of the same type are divided into three age groups, each of which is characterized by some efficiency coefficient [8]. The model includes the balance Volterra equation of the first kind with variable upper and lower limits and functional equations, describing the structure of the electricity consumption of different types of power plants. These functional equations closed the system of integral-functional equations. Also, the model includes restrictions-inequalities for the annual total growth of installed capacity. The heuristic algorithm of problem of optimizing the parameters that control the moments of removal equipment from service for two types of power plants is considered. The calculations for different economic indicators included in the goal functional are carried out.

The work is supported by the Russian Foundation for Basic Research (Project No. 15-01-01425).

Keywords: Developing system, integral model, Volterra equation.

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On Self-Regularization Properties of Difference Scheme for Linear Differential-Algebraic Equations

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ABSTRACT

Consider the problem

$$A(t)x'(t)+B(t)x(t)=f(t),\ x(0)=a,\ t\in[0,1], \tag{1}$$

where $A(t)$ and $B(t)$ are $(n\times n)$ matrices, while $f(t)$ and $x(t)$ are the given and the unknown n-dimensional vector functions, respectively. Elements of matrices $A(t)$ $B(t)$ and $f(t)$ are sufficiently smooth, and $\det A(t)\equiv 0.$ Such problems are called differential-algebraic equations (DAE). Suppose that initial data are consistent with the right-hand side.

An index is a characteristic of the complexity of problem (1). It is a minimal number of differentiations and finite transformations required for a reduction DAE to an ordinary differential equation in the normal form.

We define a uniform grid on the segment $[0,1]$ $t_l=lh,\ l=0,1,...,N,\quad h=1/N,$ and denote $A_l=A(t_l),\ B_l=B(t_l),\ \ f_l=f(t_l),\ \ x_l\approx x(t_l).$

A difference scheme

$$A_i(x_{i+1}-x_i)+hB_{i+1}x_{i+1}=hf_{i+1},\ i=0,1,...,N-1, \tag{2}$$

has been justified for DAE of the index at most 2 in the article [1].

We apply method (2) for a perturbed problem

$$A(t)\tilde{x}'(t)+B(t)\tilde{x}(t)=\tilde{f}(t),\ \tilde{x}(0)=\tilde{a}, \tag{3}$$

$$\left\|\tilde{f}_j(t)-f_j(t)\right\|_C\le\delta,\ \left|\tilde{a}_j-a_j\right|\le\delta,\quad f(t)=(f_1(t),f_2(t),....,f_n(t))^T,\ a=(a_1,a_2,....,a_n)^T,$$

$j=1,2,,...,n.$

It is shown that difference scheme (2) generates a regularization algorithm. Calculations of test examples are given.

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Keywords: differential-algebraic equations, index, ill-posed problems.

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Matrix Polynomials and Their Application

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ABSTRACT

Matrix polynomials and their application to solving integral-algebraic and higher order differential-algebraic equations are considered.

Keywords: Matrix polynomials, integral-algebraic equations, simple structure, dominant property.

The Index Preservation Property for Differentiable DAEs

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ABSTRACT

Consider systems of the form

$$\Lambda_k x := A_k(t)x^{(k)} + A_{k-1}(t)x^{(k-1)}(t) + \cdots + A_0(t)x(t) = f(t), \tag{1}$$

where $t \in T := [0,1]$, $A_i(t)$ are $n\times n$ – matrices, $i = 0,1, \ldots k$, x(t) and f(t) are the desired and the given vector-functions, correspondingly, and $x^{(j)}(t) = \left(\frac{d}{dt}\right)^j x(t)$. It is assumed that the input data is smooth enough and the following condition holds

$$\det A_k(t) = 0 \ \forall t \in T. \tag{2}$$

Such systems are commonly referred to as differential algebraic equations (DAEs). The first order DAEs, when k = 1, have been fairly well studied. In this talk, we introduce a concept of index for the DAE (1) and show that the system

$$\left(\frac{d}{dt}\right)^{(j)} \Lambda_k x = f^{(j)}, j = 1,2 \ldots$$

has the same index as the original system (1). In turn, it means that there exists such j that the sum $k+j$ is greater than the index of (1). We shall focus on the properties of the systems whose index is not greater than the order of the system.

This work has been supported by the Russian Foundation for Basic Research, Project no. 16-51-540002.

Keywords: Differential algebraic equations, index, order.

The Connection between the Value of the Functional and the Value of the Argument in the Linear Quadratic Optimal Control Problem

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ABSTRACT

Consider the quadratic functional

$$I(u) = \int_{\alpha}^{\beta} [\langle Au, u\rangle + 2\langle Bu, u\rangle + \langle Cx, x\rangle] dt, \tag{1}$$

With the connections

$$A_1 \dot{x} + A_0 x = Du, t \in T = [\alpha, \beta], \tag{2}$$

$$x(\alpha) = 0,$$

Where $A \equiv A(t)$ is an m×m – matrix, $B \equiv B(t)$ is an n×m – matrix, $C \equiv C(t)$ is an n×n – matrix, $A_1 \equiv A_1(t), A_0 \equiv A_0(t)$, is an n×n – matrices, D$\equiv D(t)$ is an n×m – matrix $x \equiv x(t)$, is n-dimensional smooth on T vector-function, $u \equiv u(t)$ is an m-dimensional vector-function, $\langle .,. \rangle$ is a scalar product in the Euclid space. It is assumed that

$$\det A(t) = 0, \qquad \det A_1(t) = 0 \ \forall \ t \in T.$$

Additionally, we suppose that the input data in (1), (2) is sufficiently smooth for further reasoning and the matrices A, C in (1) are symmetric. This talk focuses on the following issues:

1. Finding conditions that provide $I(u) \geq 0 \forall u \in \mathbf{C}^{\infty}(T)$;
2. Finding conditions under which the small value of the functional $I(u) \leq \varepsilon$, where $\varepsilon \in [0, \varepsilon_0]$, corresponds to the small value of the control $\|u(t)\|^2_{L_2(T)} \leq \kappa\varepsilon$, where κ is some constant.

This work has been supported by the Russian Foundation for Basic Research, Project no. 16-51-540002.

Keywords: Optimal control, quadratic functional, differential algebraic equations, index.

Numerical Method for Solving an Inverse Problem of Nanocrystallization

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ABSTRACT

The development of numerical methods for solving an inverse problems of nanocrystallization is of great interest [1,2]. In this contribution, the numerical method for solving an inverse problem of nanocrystallization that associate with the growth of new-phase nucleation centers in a two-component solution in the nanocrystallization of a solid amorphous alloy is considered.

We will use the following designations: t - time, x - distance from the nucleation centers to the alloy boundary. The mathematical model describing the nanocrystal growth is proposed in [3] and can be represented as follow

$$\frac{\partial c_i}{\partial t} = D_i \frac{1}{R^2 x^2} \frac{\partial}{\partial x}\left(x^2 \frac{\partial c_i}{\partial x} \right) + \frac{\partial c_i}{\partial x} \frac{R'}{R} x, \quad x > 1, \quad i = 1,2 \tag{1}$$

$$\frac{\rho\ D_i}{R} \frac{\partial c_i\left[1+0,t\right]}{\partial x} + c_i\left(1,t\right) \sum_{k=1}^{2} M_k I_k - M_i I_i = 0, \quad x > 1, \quad i = 1,2 \tag{2}$$

$$\frac{dR}{dt} = \frac{D_i}{\rho} \sum_{i=1}^{2} \left(\frac{\partial c_i\left[t,1+0\right]}{\partial x} \right) \tag{3}$$

with initial and boundary conditions

$$R(0) = R^0 + 1;\ c_i\left(x,0\right) = \beta_i\left(xR^0\right), \quad x > 1, \quad i = 1,2 \qquad 0 \le t \le T \tag{4}$$

$$\lim_{x\to\infty} c_i\left(x,t\right) = c_i^0, \quad i = 1,2. \tag{5}$$

where i is number of component $c_i\left(x,t\right)$ is concentration of corresponding component, D_i is diffusion coefficient, R is radius of growing spherical nanocrystal, ρ is density of nanocrystal , M_i molecular mass of component and I_i is scattered intensity of component. In this problem it is required to determine the function $R(t)$. To solve this problem, we replace the differential equation (1) on a finite-difference equation with an additional stabilizing functional which is multiplied on some parameter α . The computing scheme involves solving the equation for each time spatial step. We choose the discretization steps by time variable according special conditions. As a result, we achieve the stability of computational scheme.

To evaluate the stability of the method and to obtain experimental error estimates of the radius we carried out the computational experiments. We estimate the deviation of calculated radius R_α from the test radius function R_m via the quantity $\Delta = \max\limits_{t\in[0,T]} \left| R_m - R_\alpha \right|$. The computational results for the some test data is presented in the paper and confirmed the reliability and the efficiency of the proposed method.

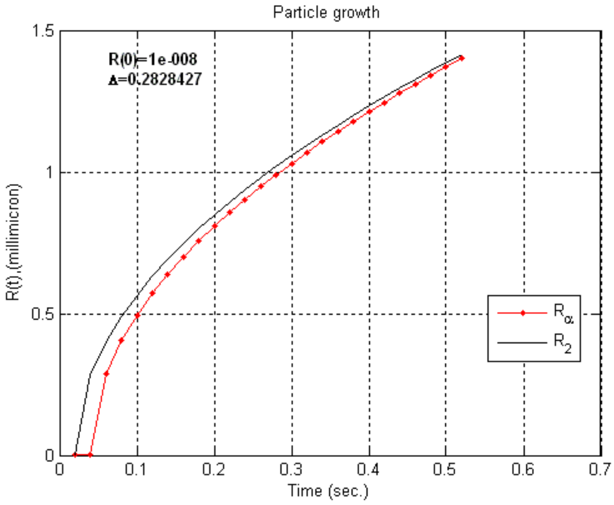

Fig.1 The radius of growing spherical nanocrystal. Designation R_m corresponds to test function, the numerical solution to problem (1)-(5) denote as R_α . The notation Δ corresponds to the quantity which is calculated by formula $\Delta = \max_{t\in[0,T]} |R_m - R_\alpha|$.

Keywords: Nanocrystallization, inverse problem, regularization method, numerical method, computational scheme.

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On the Numerical Solution of a Convolution-Type Volterra Equation of the First Kind

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ABSTRACT

The research addresses a convolution-type Volterra integral equation of the first kind whose kernel equals zero in a small neighborhood of 0 [1]. This equation was introduced in [2] to solve an inverse boundary-value problem of heat conduction. The algorithms [3, 4] based on the methods of product integration and mid-point rule were applied to find an approximate solution to the integral equation. A computational experiment demonstrates that both numerical methods have the second order mesh convergence, however the product integration method is preferable. Consideration is given to the mechanisms of error emergence in the computations with a fixed length of significand in the computer's floating-point number representation. The computer algebra system MAPLE 10 was applied to demonstrate typical cases of systematic error accumulation. To illustrate an instantaneous loss of high-order digits we applied the software [5,6] that realizes the function of tracking the valid digits of significand.

This work has been supported by the Russian Foundation for Basic Research, Project No. 15-01-01425a.

Keywords: Volterra integral equations, numerical solution, product integration method.

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Numerical Study of Two-Dimensional Advection-Diffusion Problem in an Infinite Domain

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ABSTRACT

Numerical study of advection-diffusion type equations has been an active area of research and still remains important because of its wide-range of applications in computational sciences. On the other hand, Generalized Integral Representation Method (GIRM) is designed to numerically solve Initial-Boundary Value Problems for differential equations. In this work, we develop a numerical scheme based on GIRM for evaluation of the two-dimensional problem of advective diffusion in an infinite domain. Accurate approximate solution is obtained and compared to the exact solution.

Keywords: Numerical solution of advection-diffusion problem, two-dimensional advection-diffusion equation, generalized integral representation method (GIRM), numerical schemes based on GIRM

Some Properties Of Integro Cubic Splines

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ABSTRACT

In this talk, we consider shape-preserving properties of the integro-cubic spline. The sufficient conditions for convex, monotone, or positive approximation are given. We also construct the family of monotone and convex C1 integro cubic splines. Finally, we give an application of the integro cubic spline to approximate the numerical solution of nonlinear Burger's PDE.

Keywords: Local integro spline, shape-preserving, approximation.

A general Algorithm for Constructing Local Integro Splines

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ABSTRACT

In this talk, we propose a general algorithm for constructing local integro splines. We also offer the conjecture that an integro spline $S_m(x)$ has super-convergence properties for even and odd degrees derivative. We use the proposed general algorithm to construct a local integro sextic spline and study its super-convergence properties. The orders of convergence at the knots are eight (not seven) for function, six (not five) for the second derivative and four (not three) for the fourth derivative. Numerical experiments confirm the super-convergence properties.

Keywords: Integro splines, local integro splines, super-convergence.

Numerical Solution of Integro-Algebraic Equations by Block Methods

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ABSTRACT

In the report we consider the systems of integral equations

$$A(t)x(t)+\int_0^t K(t,s)x(s)ds = f(t),\ \ 0 \le s \le t \le 1 \qquad (1)$$

here $A(t)$ and $K(t,s)-(n\times n)$ matrices, $f(t)$ is a given and $x(t)$ is an unknown n-dimensional vector-functions. Any continuous vector-function $x(t)$, which satisfies equation (1) is a solution of the problem(1).

Here the matrix $A(t)$ satisfies condition

$$\det A(t) \equiv 0 \qquad (2)$$

Sufficient conditions for the existence of a unique continuous solution to problem (1) have been formulated in [1]. Interpolation block methods for the numerical solution of a selected class of equations are proposed and are showed results of numerical solution in the report.

This work has been supported by the Russian Foundation for Basic Research, Projects No. 16-51-540002-Viet, 15-01-03228 a, 16-31-00219-mol-a.

Keywords: Integro-algebraic equations, block methods, numerical solution.

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Reducibility of Linear Homogeneous System Differential Equations with Semiperiodic Coefficients

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ABSTRACT

Property reducibility of linear system differential equations play a significant role in the research for the theory of asymptotic stability of the system of linear differential equations. In 1892, Aleksandr Mikhailovich Lyapunov proved that the property reducibility of linear homogeneous systems of differential equations with periodic coefficients. In 1992, Jorba, A. and Simó, C. proved that the Reducibility of Linear Differential Equations with Quasi-Periodic Coefficients. Moreover, all periodic functions and all Quasi-Periodic functions will be semi-periodic functions. In this paper, we consider one new definition of semi periodic functions and consider the reducibility of linear homogeneous systems of differential equations with semi periodic coefficients. Therefore, main theorem this paper will be generalized results of the theorem of Lyapunov.

Keywords: Reducible systems of linear differential equations, integral matrix of system linear differential equations, complex continuous function, fundamental matrix.

Approaches for Determining Index of Partial Differential Algebraic Equations

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ABSTRACT

Consider the system of partial differential equations with constant coefficient matrices

$$\Lambda(D_t, D_x)u := AD_t u + BD_x u + Cu = f(x,t), (x,t) \in R^2, \quad (1)$$

where A, B, C are constant $n \times n$ matrices, $D_t \equiv \partial / \partial t, D_x \equiv \partial / \partial x$; $f(x,t)$ is a given vector function, and $u \equiv u(x,t)$ is the sought vector function. We study the initial–boundary value problem for system (1) with conditions of the form

$$u(x_0, t) = \psi(t),\ U(x, t_0) = \phi(x),\ (x,t) \in U = X \times T \subseteq R^2, \quad (2)$$

where $X = [x_0, x_1], T = [t_0, t_1]$. Additionally, the vector functions $f(x,t), \psi(t), \phi(x)$ are assumed to be sufficiently smooth.

By the solution of system (1) in the domain U we mean any vector function $u(x,t) \in \mathrm{C}^1$ (U) that satisfies equation (1) identically in U. The following forms of singularity are admitted:

$$\det A = 0, \det B = 0, \det[\lambda A + B] = 0\ \forall \lambda \in \mathrm{C}, \quad (3)$$

where λ is a scalar (generally complex-valued) parameter. Systems (1) satisfying conditions (3) are called singular systems or systems of non-Cauchy-Kovalevski type or partial differential algebraic equations. The analysis of ordinary differential algebraic equations has revealed the existence of the integral characteristics of the system, called its *index*. The index determines the order of the derivative of the input data, on which depends the solution of the problem. From this arises the question: how to determine the index of partial differential algebraic equations? In the literature we can find a number of definitions which are not reducible to each other, and our task is to select the most appropriate definition. We have constructed the operator, which reduces the original system to a system of integro-differential equations, solved for the derivative with respect to *t* (and to *x*). If at some step of the process *l* matrix $A^l(\mu)$ is regular, then we say that index of the system (1) with respect to *t* is equal to *l*. In further studies, we intend to present full results of research approach for determining index of partial differential algebraic equations (1).

Keywords: Partial differential algebraic equations, regular, index, singular systems.

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The Necessary and Sufficient Convergence Conditions for Some Two and Three−Point Newton's Type Iterations

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ABSTRACT

In this talk we give the necessary and sufficient conditions for two and three−point iterative methods to be p−order ($2 \le p \le 8$) of convergence. We also find optimal choices of iteration parameters. These conditions can be effectively used to establish the convergence of some methods. In particular, the convergence order of some known optimal order methods are verified using the proposed sufficient convergence criterion.

Keywords: Nonlinear equations, Newton's type iterations, convergence order optimal order.

Necessary Optimality Conditions for Impulsive Control Problems with Intermediate State Constraints

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ABSTRACT

This report deals with an impulsive optimal control problem with trajectories of bounded variation and impulsive controls (regular vector measures). The problem under consideration contains general terminal and intermediate state constraints on the one-sided limits of trajectories of bounded variation. The nonsmooth Maximum Principle for such problem were obtained in [1] under the assumption that the examined measure has no the continuous singular component. In this report we consider the general case.

Keywords: Optimal control, optimality conditions, Maximum Principle.

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Multistep Method for Solving Singular Integral-Differential Equations

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ABSTRACT

We consider the linear integral-differential equations of the first order with the identically singular matrix at the derivative. For these systems, the initial conditions is given and assumed consistent with the right part. Considered tasking in this paper arise in the mathematical modeling of complex electric circuits. By using the apparatus of matrix polynomials a class of problems, which having a unique solution, is marked out. The difficulties of the numerical solution of such problems, in particular the instability of many implicit methods is considered. For numerical solution of this class of problems we have suggested multistep methods, which are based on an explicit quadrature formula for the integral term Adams and extrapolation formulas. Sufficient conditions for the convergence of such algorithms to the exact solution is formulated.

Keywords: Integral differential equation, numerical methods, mathematical modeling, electric networks.

About Implicit Multistep Methods for Numerical Solution of Integral-Algebraic Equations

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ABSTRACT

In the report we considered systems of integral equations:

$$A(t)x(t) + \int_0^t K(t,s)x(s)ds = f(t),\ 0 \le s \le t \le 1,$$

where $A(t)$ and $K(t,s)$ are $(n \times n)$ matrices, while $f(t)$ and $x(t)$ are the given and the unknown n-dimensional vector functions, respectively. And $A(t)$ is singular matrix:

$$\det A(t) \equiv 0.$$

These problems are called integral-algebraic equations (IAEs) with smooth kernels. For numerical solution of IAEs with smooth kernels we constructed implicit multistep methods:

$$A_{i+1}x_{i+1} + h\sum_{l=0}^{i+1} \omega_{i+1,l} K_{i+1,l} x_l = f_i.$$

Standard multistep methods are unstable for the problem.We present conditions on coefficients $\omega_{i+1,l}$, results numerical experiments and regions of stability.

This work has been supported by the Russian Foundation for Basic Research, Projects No. 16-51-540002-Viet, 15-01-03228 a, 16-31-00219-mol-a.

Keywords: Implicit multistep methods, numerical solution, integral-algebraic equations.

A New Development of Immersed Finite Element Methods

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ABSTRACT

We give a survey of new class of discretization methods, called IFEM, for elliptic interface problems using structured grids then present a new development also. For scalar problems we consider using Lagrangian type P_1, Crouzeix-Raviart nonconforming P_1, bilinear and Rannacher Turek element for rectangle grids. We point out that the earlier version of IFEMs does not yield optimal order of convergence. As a remedy for this, we consider two variants. One is to add line integrals to the bilinear forms to make the scheme consistent. Another is to us CR(or RT) nonconforming $P_1(Q_1)$ basis functions. We note that the convergence of IFEM with nonconforming elements are guaranteed with an additional regularity assumption that Darcy velocity belongs to H^1. Applications to mixed methods, and multigrid convergence is also discussed.We also discuss the IFEM for elasticity problems, using CR nonconforming P_1 basis functions. In this case, the bilinear forms are modified by adding the stability terms to guarantee the Korn's inequality. Several numerical examples are provided which support the theory.

Keywords: Immersed Finite element method, interface problem, nonconforming discontinuous Galerkin elasticity equation.

A Method for Solving Boundary Value Problems for Linear Differential-Algebraic Equations with Perturbations in the Form of Fredholm Integral Operators

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ABSTRACT

We consider the system of linear differential-algebraic equations

$$A(t)\dot{x}(t)+B(t)x(t)+\lambda\int_{\alpha}^{\beta}K(t,s)x(s)ds=f,\qquad t\in T=[\alpha,\beta], \tag{1}$$

where $A(t),B(t),K(t,s)-(n\times n)$-matrix, $\dot{x}(t)=dx(t)/dt,\ x(t),\ f\equiv f(t)-$ the unknown and the given vector function, respectively, with the boundary conditions:

$$Cx(\alpha)+Dx(\beta)=a, \tag{2}$$

where $C,D-(m\times n)$-matrix, a - a given vector. It is assumed that the input data are sufficiently smooth and the following condition holds

$$\det A(t)\equiv 0,\ t\in T. \tag{3}$$

Problems (1), (2) with the condition (3), often occur in the analysis of complex electrical and electronic circuits. In this report, we talk about the convergence conditions for the least squares method and present the results of numerical experiments.

Keywords: Differential equations, boundary value problems, differential algebraic equations, index, least squares method.

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On a Model of Thermoviscoelastic Waves

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ABSTRACT

Suppose Ω is a domain of $\mathbb{R}^2$with a boundary $\partial\Omega$ of C^∞class, and $t > 0$. Let us consider the system of partial differential equations [1]

$$u_{tt} - h\Delta u_{tt} + \Delta^2 u + \alpha\Omega\theta - \int_0^t g(t-\tau)\Delta^2 u(\tau, x, y)d\tau = 0,$$

$$\theta_t - k\Lambda\theta + \gamma\theta - \alpha\Delta u_t = 0,$$

which describes small transverse vibrations of homogeneous isotropic thermoviscoelastic plate of uniform thickness h. Functions $u = u(t, x, y)$ and $\theta = \theta(t, x, y)$ on the cylinder $(0; +\infty) \times \Omega$ define transverse deflection of plate and heat distribution, respectively; real parameters α, γ, k are echanical and thermal characteristics; function $g = g(t)$ characterizes the rheological properties of the material (creep); $\Delta = \partial_{xx}^2 + \partial_{yy}^2$ is the Laplace operator. Excluding the value of θ, we obtain the integro-differential equation

$$(1 - h\Delta)u_{ttt} - (k\Delta - \gamma)(1 - h\Delta)u_{tt} + (\alpha^2 + 1)\Delta^2 u_t - (g(0) + k\Delta - \gamma)\Delta^2 u -$$
$$- \int_0^t [g'(t-\tau)\Delta^2 - g(t-\tau)(k\Delta^3 - \gamma\Delta^2]u(\tau, x, y)d\tau = 0.$$

Then we consider the problem with initial and boundary condition

$$u|_{t=0} = u_0(x, y), u_t|_{t=0} = u_1(x, y), u_{tt}|_{t=0} = u_2(x, y); u|_{(x,y)\in\Omega} = 0.$$

The unique solvability of this initial boundary value problem is studied on the basis of its reduction to the abstract integro-differential equation [2]

$$Bu^{(N)}(t) - \sum_{i=1}^{N} A_{N-i}u^{(N-i)}(t) - \int_0^t k(t-s)u(s)ds = f(t), \quad u^{(i-1)}(0) = u_{i-1}, i = 1, \dots, N,$$

where $B, A_{N-i}, k(t): E_1 \to E_2$ are closed linear operator and B is Fredholm operator, E_1, E_2 are Banach spaces.

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Keywords:

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Numerical Solution to an Optimal Control for Hybrid Systems

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ABSTRACT

An optimal control problem for a hybrid system is considered. The hybrid dynamics is generated by a finite collection of dynamical systems in the following way. At each time moment the state variable is driven by a system from the collection. Such system is called active. There are two types of admissible control actions: 1) if certain conditions are fulfilled, an active system can be switched to another one, 2) the dynamics of active systems can be controlled in the common way. In our approach, we split the optimal control problem in two subproblems. One of them is a nonlinear optimization problem (to determine optimal switching moments), while the other is an optimal control problem with intermediate state constraints (to describe an optimal dynamics of active systems). The latter can be reduced to a classical optimal control problem by a specific penalty method. This allows us to use a combination of standard algorithms to solve the problem in question.

Keywords: Optimal control, hybrid systems, dynamical systems.

Numerical Solution of Integrodifferential-Algebraic Equations

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ABSTRACT

Numerical solution of boundary value problem for nonlinear system of integrodifferential-algebraic equations is considered. Method of solution continuation with respect to the best parameter and shooting method are applied. Test examples demonstrate some advantages of the approach.

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Keywords: Nonlinear equations, differential equations, boundary value problems.